

# Tensor model of IMS network

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**Abstract**—IMS (IP multimedia subsystem) is a main trend of development NGN. The base of IMS is an Internet Protocol which initially did not provide support for quality of service. However, for the transmission of multimedia streams is necessary to ensure a given level of quality of service (QoS). Therefore, IMS QoS estimation is a necessary task but a high complexity of network topologies and a large number of information flows makes it difficult to solve this problem. In IMS network, QoS support is provided by the interaction of systems of IMS on control and application planes. This interaction based on the signalling message exchange that also has delay.

This paper presents the results to create a nodal tensor model of IMS network for delay analysis. In this model, the main systems of the IMS network such as CSCF (Call Session Control Function), HSS (Home Subscriber Server) etc. are represented as a single-server queuing systems. Model is formed in accordance to the postulates of the tensor analysis of networks. Moreover, proposed model allow estimating the delay time as for the individual routes, and for the whole network. Research of the model allowed determining such distribution of traffic that resulted to the minimum value of the delay that in turn leads to increased QoS of information flows in an IMS network.

## I. INTRODUCTION

The basis of the IMS network is Internet Protocol (IP) which is most used protocol in modern communications. IP actually has become a major network layer protocol. Moreover, investigation of probability and time characteristics of IP networks is one of topical tasks of communications development because these characteristics determine Quality of Service (QoS) in IP networks. Main QoS parameters are defined as: IP packet loss ratio, IP Packet Transfer Delay, IP packet delay variation and IP packet error ratio [1], [2]. These characteristics define service level for different types of information streams for the purpose of optimal network resources distribution. In terms of queuing theory, IP packet loss ratio is a probability of packet loss and IP Packet Transfer Delay can be considered as mean time delay.

In IMS network, QoS support is provided by the interaction of a systems of IMS on control and application planes. This interaction based on the specific scenarios of the signalling message exchange [3]. These processes also have a delay. Thus, the task of analysis of IMS network can be reduced to estimation of probability and time characteristics of queuing network as model of IMS.

However, solution of this task for IMS is complicated due some factors, such as: dynamic changing structure of networks, large number of information and signalling transmission paths, dynamic distribution of network node resources,

heterogeneous information etc., and, in some cases, this task solution is next to impossible with use of classical methods of queuing systems and networks [2].

It should be noted that this task is most topical for modern infocommunication networks, but its solution is associated with some difficulties such as the problem statement, source data choice etc. including high computing costs. Difficulty of this problem solution for global networks increases depends on technologies of infocommunication networks, network topologies of various levels complexity and QoS guaranteeing for heterogeneous information streams.

This work presents how tensor analysis can be used for delay estimation of queuing networks as model of IMS. In this case, queuing networks is regarded as geometric objects whose projections in different coordinate systems are distinguish physical properties being unchanged. The result of work is a nodal tensor model for a signalling stream distribution in IMS network.

Section II outlines the base of tensor analysis of networks in relation to electrical networks. Section III presents geometrical meaning on the coordinates transformation in the tensor analysis of networks. Section IV describes the IMS system models which we can use for tensor analysis. Section V presents transformation the IMS system models to nodal form for further research and the basis of the tensor concept in the communications. The numerical results in section VI show the use of the tensor analysis to delay estimation in IMS network. Also, here is a description of the original software for a network characteristic estimation. Conclusion is in section VII.

## II. THE BASE OF TENSOR ANALYSIS OF NETWORKS

American scientist and engineer G.Kron was founder of tensor concept of system analysis who first used tensor analysis and topology for theory of electrical networks [4]. Further developments of tensor analysis ideas got in works by Happ H.H. [5], Petrov A.E. [6] and others.

Kron's theory fundamental consists of two generalization postulates. The main of the first postulate is that integration of elements does not introduce new physical properties which are not observed in primitive element of a complex system. Equation of complex system behavior corresponds to equation of primitive element state but in a matrix form. The second postulate defines that any structure changes complex system (new physical properties not bring in system; number of elements not changed but connections between elements are

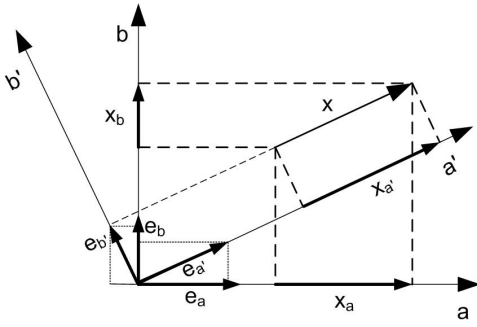


Fig. 1. Vector  $x$  transformation in rectangular coordinate systems

change) bring to the effect that new system will be described by same set of matrix equations but with other position of the elements in matrix. Transition from the old system to the new can be realized with transformation matrix.

According to G.Kron, electrical networks are described as mesh (close-path), junction (open-path) and orthogonal (mixed type) networks. He used the following matrix equation for networks investigation:  $C^T e = C^T z C i'$  ( $C$  - transformation matrix,  $e$  - voltages vector,  $i'$  - loop currents vector,  $z$  - impedances matrix); junction networks  $A^T i = A^T y A e'$  ( $A$  - transformation matrix,  $i$  - branch currents vector,  $e'$  - vector of potential differences,  $y$  - conductivities matrix). These equations determine distribution of currents or voltages in electrical networks without limitations on scale and network elements connections structure.

### III. THE COORDINATES TRANSFORMATION IN THE TENSOR ANALYSIS OF NETWORKS

The basis of tensor analysis is geometrical view of investigation parameter that is geometrical interpretation of interconnection of network elements characteristics.

Characteristics analysis (which is considered at this work as mean delay but can be used others) can be determined with using geometric invariant of these characteristics in such coordinate system where the estimation of these parameters will be simpler. Geometrical interpretation of tensor analysis is presented on Fig.1 which demonstrates the main idea of the tensor concept.

In Fig.1 vector  $x$  is presented in rectangular coordinate system  $ab$  and consequently, we can determine vector coordinates and vector absolute value with the given vector projections on axes  $a$  and  $b$ . Therefore, we can write for this vector:  $x = x_a + x_b$  or

$$x = \frac{|x_a|}{|e_a|} e_a + \frac{|x_b|}{|e_b|} e_b \quad (1)$$

Then we introduce new coordinate system  $a'b'$ . Vector view in new coordinate system can be found similarly (1):  $x = x_{a'} + x_{b'}$  or

$$x = \frac{|x_{a'}|}{|e_{a'}|} e_{a'} + \frac{|x_{b'}|}{|e_{b'}|} e_{b'} \quad (2)$$

Unit vectors in new coordinate system are  $e_{a'}$  and  $e_{b'}$ . These vectors can be found from old system vectors  $e_a$  and  $e_b$  as:

$$e_{a'} = \frac{|e_{a'a}|}{|e_a|} e_a + \frac{|e_{a'b}|}{|e_b|} e_b; e_{b'} = \frac{|e_{b'a}|}{|e_a|} e_a + \frac{|e_{b'b}|}{|e_b|} e_b \quad (3)$$

Consequently, basis in new system can be defined as:  $e_{a'} = c_{11} e_a + c_{12} e_b$  and  $e_{b'} = c_{21} e_a + c_{22} e_b$  with replacements introduced in (3). Thus, we can present (3) in a matrix form, as:

$$\begin{pmatrix} e_{a'} \\ e_{b'} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} e_a \\ e_b \end{pmatrix} \quad (4)$$

or, in more general form:  $e' = C e$ .

Therefore, according to identity of expressions (1) and (2) vector  $x$  can be defined with the account taken of (3) and (4):

$$\begin{pmatrix} \frac{|x_a|}{|e_a|} \\ \frac{|x_b|}{|e_b|} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{21} \\ c_{12} & c_{22} \end{pmatrix} \begin{pmatrix} \frac{|x_{a'}|}{|e_{a'}|} \\ \frac{|x_{b'}|}{|e_{b'}|} \end{pmatrix} \quad (5)$$

Thus, expression (5) defines vector  $x$  in old coordinate system from coordinates in new system:  $x = C^T x'$  ( $C$  - transformation matrix definite interconnections between different geometrical presentations), or, also, new coordinates from old:  $x' = (C^T)^{-1} x$ . Therefore, we can come to the conclusion about the possibility of researching values projections in different coordinate systems with using information about values in one coordinate system and finding interconnection matrix (transformation matrix) for transition to other coordinate system.

### IV. THE IMS SYSTEM MODELS

According to IMS architecture, network elements interaction occurs on the following planes: application plane, signalling plane and media plane. Besides, Open Systems Interconnection Basic Reference Model defines the levels of information transmission: physical level (the interface level), data link level (the network node interconnection structure is defined), network level (the transmission paths are set for different information streams: voice, video, data etc.) Thus, information and signalling flows transmission is provided at several planes and levels and there are different structures between the elements on each of them. Moreover, it takes into account that IMS flows depend on network procedures which are based on SIP scenarios.

Therefore, an IMS service model should consist of a few phases of service providing: request of services (information transmission paths are set), service providing (information is transmitted both from subscriber to network as from network to subscriber), termination. Consequently, we can determine some different flow distributions and network structures which, in aggregate, determine a common model of information and signalling stream processing in IMS network.

The main elements of IMS network are: control function objects (CSCF - Call Session Control Function); servers of application, presence and database management system including

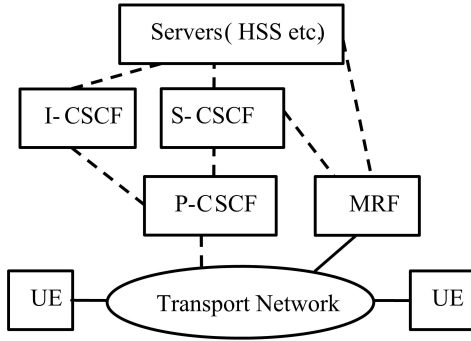


Fig. 2. IMS network architecture

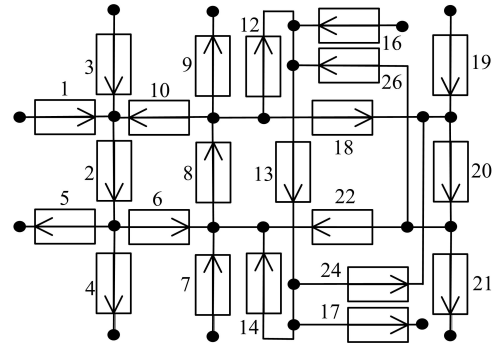


Fig. 3. IMS network model

HSS (Home Subscriber Server); resource distribution function objects (MRF-Media Resource Function): controlling MRFC (MRF Controller) and processing MRFP (MRF Processor). A simplified network architecture is presented in Fig.2.

Control function objects are P-CSCF (Proxy-CSCF), I-CSCF (Interrogating-CSCF) and S-CSCF (Serving-CSCF). The P-CSCF is the first IMS system on signalling plane which provides access of user equipment (UE) to the IMS network. It forward messages from UE to other systems of signalling plane (and conversely) and also performs some other functions. I-CSCF is a communication point within an IMS network for connections to a subscriber of this network. S-CSCF is a central system of IMS network which responsible for storing the service profiles of users and assigned for subscribers registration, routing and session maintaining. CSCF systems exchange information among themselves and also I-CSCF, S-CSCF exchange signalling messages with HSS, MRF etc.

The IMS systems interaction is based on the SIP message exchange. This process is well modelled as an arrival service in queuing network. The queuing network structure (as a network model) has a various topologies in dependence from IMS plane. Any system of IMS can be presented as a queuing network and every system of this network is a model of an interface (for physical level) of system or a direction (for logical level) of the information transmission.

This interface or direction is presented as a separate queuing system (QS) with delay, in some cases with limited buffer. Thus, taking into account the distribution of signalling flows the IMS model can be presented as queuing network whose topology is shown in Fig.3.

In the case of the signalling flows distribution, it is necessary take into account transmission routes and not only a physical connections between a network systems are considered. Model of every route is a separate QS. Type of QS depends on service procedure of a real information processing. The main probability-time characteristics of IMS network can be determined. It should be noted that this characteristics are presented as a function of load:  $p_{loss} = f(\rho)$ ,  $T_{delay} = f(\rho)$ .

In Fig.3 information processing on network level model is presented for the IMS network. QS1 is a model of a flow from the UE and conversely QS5 is a model of a flow to the UE. The P-CSCF service process model consist of QS2, QS3 and

QS4: QS2 is a main element of P-CSCF processing, QS3 is a request generator from P-CSCF and QS4 is a message receiver to P-CSCF. Similarly I-CSCF is presented as QS7, QS8, QS9 and QS13, QS16, QS17 are a model of HSS; the model of S-CSCF consist of QS19, QS20, QS21. Queueing systems QS6 and QS10 set the interaction between I-CSCF and P-CSCF: QS6 is a flow from P-CSCF to I-CSCF and QS10 is a stream in the other direction between these elements of IMS. In the same way QS12 and QS14 are a flows between I-CSCF and HSS. Similarly QS18, QS22 and QS24, QS26 are the streams between I-CSCF & S-CSCF and S-CSCF & HSS respectively. The numbering of the queueing systems is determined by the formation of the tensor model.

Each queueing system serves the streams with delay and in some cases with loss. Then we can found time that necessary for different procedures in IMS network as a delay in queuing network. Thus, this characteristic can be determined as  $T_{delay} \approx \sum_{i=1}^m T_{delay,i}$ , where  $m$  is a number of systems which compose transmission/processing path (if necessary, alternative routes can be used). Besides, probability of loss can be found for any route as  $p_{loss} \approx \sum_{i=1}^m p_{loss,i}$  (in low losses). It is natural that information and signalling traffic distribution in IMS is not random but correspond to process of IMS network interaction.

## V. THE TENSOR CONCEPT IN THE COMMUNICATIONS

In this work the method of IMS network analysis is based on the following assumptions [7]–[9]. First, a streams with a specific arrival intensity ( $\lambda$ ) will cause to the same load ( $\rho$ ) in the network systems when the structure of network being changed but a service intensity is constant, and, consequently, relationship (invariant) can be define as [8], [9]:

$$\rho\lambda = \rho'\lambda' \quad (6)$$

where variables with apostrophe are used for a one network structure, a variables without apostrophe are used for another structure. Secondly, the interaction of systems in network do not change anything in the service process of streams, thus analysis of a complex system (or network) consists in the

identification of a primitive element and determination of properties of this element, and further, an algorithm of analysis is carried forward to any complex system (or network). Thirdly, the network structure change is not supposed to result in qualitative changes of the main relationships of a primitive element, but causes to quantitative changes.

The nodal method of the tensor analysis is more appropriate for the IMS network research that can be explain by the structure of model of IMS network. In the previous work [9], the correspondence between the system utilization in initial network  $\rho$  and the system utilization in primitive network  $\rho'$  was set by the use of the transfer matrix  $A$ :  $\rho' = A\rho$ , where matrix  $A$  is defined by the relation between the "nodal" intensities and the intensities of the branches. Thus, we can get from (6)  $A\rho\lambda' = \rho\lambda$  by means of the known relationship for load:  $\lambda = \mu\rho$ , where  $\lambda$  - the arrival intensity,  $\mu$  - the service intensity. Further, the relationship between intensity in initial ( $\lambda$ ) and primitive ( $\lambda'$ ) network is determined as:  $\lambda' = (A^T)^{-1}\lambda$ . Consequently, we can get  $(A^T)^{-1}\lambda = \mu'A\rho$ . Further,  $\lambda = A^T\mu'A\rho$ . Finally, the equation for the nodal method is:

$$(A^T\mu'A)\rho = A^T\lambda' \quad (7)$$

Equation (7) is solved for variable  $\rho$ . Thereafter, the loads of systems  $\rho$  of the initial network are determined as:  $\rho_{branch} = A\rho$ . Then, the type of queuing systems (M/M/1, M/D/1, M/M/s/N, M/D/1/N etc.) is chosen for further analysis. It allows estimating the required characteristics, for example: the procedure delay or the delay of all network is:  $T = \sum_{i=1}^{\alpha} T_i$ , where  $\alpha$  - number of the network nodes;  $T_i = f(\rho)$  - the average delay of the  $i$ -th system. Also, the stream intensity can be find as:  $\lambda_{branch} = \mu_{branch}\rho_{branch}$ . Consequently, nodal method of tensor analysis can be use for load balancing in IMS networks too.

This method makes it possible to the load estimation with a low computational costs, and, also, the main network characteristics estimation is get: the state probability distribution:  $p_n = f(\rho)$ , the average queue  $\bar{N}$  and the average delay  $\bar{T}$ . For IMS network, the solution of (7) provides to obtain the next characteristics: the packet loss probability, the delay (and its deviation) and the network throughput.

## VI. THE NUMERICAL RESULTS

According to Fig.3, the model of the processing of flows in IMS is a queuing network where the queuing systems are a models of the P-CSCF, I-CSCF, S-CSCF and HSS service processes.

This model should be transformed to the nodal form (Fig.4): model is not fully shown because of the great saturation of information. The load of each system in this model is defined as the ratio between the nodal loads that are shown in the Fig.4 dashed lines. Connections are broken if meshes occur but, in further analysis, the arrival intensity equality is taken to account in corresponding branches: it is demonstrated by the example of the mesh QS2-QS6-QS8-QS10. In that

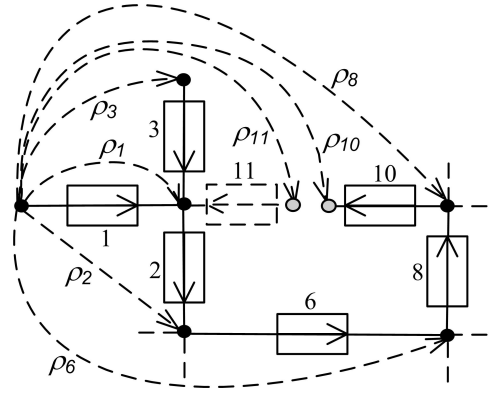


Fig. 4. Nodal model of IMS network (fragment)

case, imaginary queuing system QS11 is introduced and the parameters of QS11 are equal to the parameters of QS10 ( $\lambda_{11} = \lambda_{10}$ ,  $\mu_{11} = \mu_{10}$ ). Similarly, the imaginary QS15, QS23, QS25 and QS27 are introduced.

Besides, it's necessary to define of the service rate for each queuing systems. This parameter is determined by the capabilities of devices on a logical and physical level because on a logical enough to specify the intensity of messages but on the physical length of the messages is necessary to consider too.

According to [9], the initial and primitive network load equivalence is determined and the matrix  $A$  is got. The matrix  $A$  is not presented here because it has high dimensionality and sparseness: for example, the first column of matrix  $A$  is shown below. In this case, the first column corresponds to a first nodal load (Fig.4) and so  $\lambda_1$ ,  $\lambda_3$  and  $\lambda_{11}$  is incoming intensities and  $\lambda_2$  is outgoing stream. Therefore, the first column elements of matrix  $A$  has such values.



$$A^T \lambda' = \begin{pmatrix} 0 \\ 0 \\ -\lambda_3 \\ \lambda_4 \\ \lambda_5 \\ 0 \\ -\lambda_7 \\ 0 \\ \lambda_9 \\ \lambda_{10} \\ -\lambda_{10} \\ 0 \\ 0 \\ \lambda_{14} \\ -\lambda_{14} \\ -\lambda_{16} \\ \lambda_{17} \\ 0 \\ -\lambda_{19} \\ 0 \\ \lambda_{21} \\ \lambda_{22} \\ -\lambda_{22} \\ \lambda_{24} \\ -\lambda_{24} \\ \lambda_{26} \\ -\lambda_{26} \end{pmatrix} \quad (9)$$

In accordance to (9), intensities of request as initial data for calculation will only intensities that attending in this equation. Moreover, these intensities are determined by the traffic of network. As an example,  $\lambda_{19}$  is a traffic from S-CSCF which is specified by an amount messages formed in accordance with the scenario of interaction. According to [3], during the user registration S-CSCF sends the messages: "unauthorized", OK, MAR and SAR and for a complete analysis should take into account all messages in all scenarios across systems.

For further analysis we should be choose the type of queueing systems: in this work it is M/M/1 queueing system but it can be other type [9]. Thereby, we finally determined the full set of input data for analysis of IMS network model: intensities of service, arrival intensities and type of queueing systems.

The obtained data are substituted into equation (7) that is solved for the  $\rho$ . Further, the loads of all queueing systems  $\rho_i$  are determined as  $\rho_{node} = A\rho$ . Subsequently, values of loads are used for calculation of delay of each queueing systems in accordance with the chosen type of system (M/M/1). Thus,

the queueing system delay is determined as  $T_i = \frac{1/\mu_i}{1-\rho_i}$  and the procedure delay is a sum of delays in systems that is participated in this service process:  $T = \sum_i \frac{1/\mu_i}{1-\rho_i}$  (the imaginary branches are not to taking into account). As an example, the procedure of user registration in network is considered. In according to scenario of user registration [3], delay of this procedure presents as:  $4T_{P-CSCF} + 6T_{I-CSCF} + 4T_{HSS} + 4T_{S-CSCF}$  and this let concluded that I-CSCF adds the largest contribution in total delay of this procedure. In relation to IMS model, the total delay of registration will be presented as:

$$2(T_1 + 4T_2 + 2T_3 + 2T_4 + T_5 + T_6 + 3T_7 + 6T_8 + 3T_9 + T_{10} + T_{12} + 4T_{13} + T_{14} + 2T_{16} + 2T_{17} + T_{18} + 2T_{19} + 4T_{20} + 2T_{21} + T_{22} + T_{24} + T_{26}) \quad (10)$$

In the same way, we can define the expressions for the time characteristics of other scenarios of IMS. Also, the total network delay can be determine as the sum of delays for all systems (without the imaginary system delays):  $T = \sum_{i=1}^{27} \frac{1/\mu_i}{1-\rho_i}$ .

The research of the tensor model of IMS network revealed that the most loaded element is the central system of I-CSCF (QS8) because the most signalling messages arrives to this system. Consequently, the service intensity of QS8 should be more then for other systems to avoid overloading. Then, the service rates are  $\mu_i = 10$  but  $\mu_i = 20$  for QS8. The arrival intensities  $\lambda_i$  was set to values not exceeding 6 ( $\lambda_7, \lambda_9$ ). In this case, the delay of user registration is 11.462 conditional units and the total network delay is 4,377 conditional units. And the maximum traffic is served in the QS8 ( $\lambda_9 = 12$ ), QS2 ( $\lambda_2 = 8$ ), QS13 ( $\lambda_{13} = 8$ ), QS20 ( $\lambda_{20} = 8$ ). In other systems, the traffic does not exceed 6. For calculation, the original software [9] used.

#### A. The software for a network characteristic estimation

The nodal method of tensor analysis of network used for calculation in the applied software [9]. The base algorithm of software is presented in Fig.5. This software has a two main work blocks: "Model" and "Analysis".

The "Model" block is a field for network topology drawing. The "Analysis" is designed for data input and calculation. This block contains several tabs for input and output data. The matrix  $A$  displays in the tab "Transfer matrix". The matrices of service and arrival are calculated in according with (7) and are displayed in separate tabs. The "Data input" tab is used for a system parameter setting: intensities of service and arrival, queueing system type, buffer size, order of distribution, route and format for result. The tab of results required for data output both for every system and for a whole network: the stationary probabilities (for the systems), the average queue, the average delay, the nodal and system loads, the arrival intensity for every system.

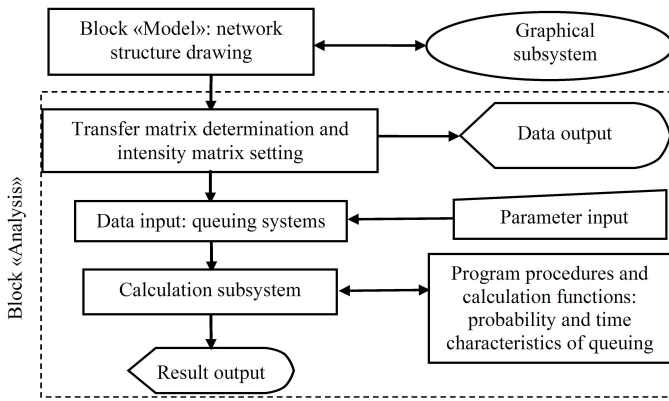


Fig. 5. The base algorithm of the program for a network characteristic estimation

## VII. CONCLUSION

The efficient allocation of IMS network resource management is complicated because it demands of control to many systems and processing the considerable amount streams. Tensor analysis of networks provide opportunities to take into account the process-structure interaction and it has the high application flexibility that result in the compute cost decreasing, the delay reducing for dynamic management infocommunication systems and the good network scalability. Moreover, the tensor model of IMS network allows to estimate the different characteristics: delay, probability, loss etc. In further, the obtained results can be used to optimize of IMS network for procedure delay reducing to improve the level of service. Besides, the tensor method let to formalize in a simple way for the IMS network computer-aided design that is result in increase of the QoS of information streams in IMS networks and allows to provide more evenly the network traffic distribution.

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